Page 61: we introduce time variable and probability of a particular network realization **X** at time t given by the distribution ., Temporal evolution of the probability distribution is expressed in the form of a master equation, which is a linear differential equation for the probability that any network, owing to the microscopic dynamics, is in the configuration **X**. In this case a stochastic description is applied by introducing the rates that expresses the transition from realization **X** to realization **Y**. if we assume that the process has no time memory, we have a Markovian process, and the temporal change of obeys a master equation of the form

This is normally distributed. **Note:** If P(Y, t) is some optimal state, could we do the model that way?

* The state of the network is the set of all the nodes

Master Equation:

Constraint:

are the transition weights and carry the unit

They can be simplified by considering that the change of sate of a node is determined only by the local interaction with the nodes directly connected to it. **THIS IS RELEVANT.** In the case where the local dynamics have the same parameters for all nodes, the transition rates can be simplified and read

Idea: interactions are a Poisson Process (rate to be determined from the data), and there’s a certain probability that a given interaction is with a successful forager. (Encountering a successful forager is also a Poisson process – the rate will be different depending on degrees of success). So, if the nodes that it’s interacting with are foragers, then they’ll leave. They’ll be interacting with ants either way, though, right?

Average number of items in state A:

Mean-field theory assumes that everything is homogeneous (i.e. all of the nodes are following the same rules), so we can write down equations like

Where 1, 2, …, k indext he possible states of each individual node. These equations concern average values and are deterministic. The explicit form of the functions depends on the specific interactions among the nodes, the transition rates, and the number of allowed microstates.

If we relax the assumption that nodes are homogenous, we end up with agent-based modeling.

The equilibrium distribution is given by the Bolzmann-Gibbs distribution:

where T is temperature, is the Boltzmann factor that proves the correct dimensional units, and is the system’s Hamiltonian which expresses the energy associated with each configuration of the system. The partition function Z is the normalization factor obtained by the condition and reads – i.e. in equilibrium physical systems there is no need to solve the ME and the stationary properties of the system may be obtained by knowing the system’s Hamiltonian.

It is possible to find transition rates that drive the system towards the equilibrium by imposing the detailed balance condition on the ME:

In other words, l the net probability current between pairs of configurations is zero when . It implies that each pair of terms in the ME has a null contribution. This is *not* true for systems out of equilibrium.

x'=Ax where A is the adjacency matrix.

If

Results in

then doesn't this mean that SIR models are basically dynamical systems?

Fun fact:

Has a solution: